

IMPACT OF UNCERTAINTY ON THE PREDICTION OF AIRSPACE COMPLEXITY OF CONGESTED SECTORS

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Abstract

The ability of traffic controllers to separate aircraft determines the capacity of the region of airspace under their control, referred to as a sector. Complexity metrics, specifically dynamic density, is used as an estimate for controller workload. The prediction of dynamic density is required for the development of efficient long-term air traffic plans. This paper explores the influence of trajectory errors on the prediction of dynamic density and uses a worst-case analysis to describe the conditions under which forecast errors may lead to excessive complexity. Although the approach has general applicability, it is described using one definition of complexity. Depending on the sector and the complexity function, when a sector is highly congested, the method identifies aircraft entering the sector at certain locations, boundaries and altitudes, whose errors in prediction contribute significantly to the increase in workload. If these errors cannot be reduced, it may be necessary to limit the traffic approaching the sector from these altitudes and boundaries.

Introduction

Air traffic controllers have the responsibility of separating aircraft operating within a region of airspace under their control, referred to as a sector. The sectors are designed such that controllers are able to handle the usual flow of traffic. Various Traffic Flow Management (TFM) techniques are used to increase flow efficiency while maintaining the workload of the controllers such that safety remains uncompromised. Currently, strategic predictions of sector aircraft count are compared to the established aircraft count threshold, referred to as the Monitor Alert Parameter (MAP). MAP values do not adequately represent the level of difficulty experienced by the controllers under different traffic conditions. It has been recognized that workload should be based not only on the number of aircraft, but also their relation to each other and the

airspace geometry [1]. This concept, which includes both the intrinsic nature of traffic (airspace complexity) and human factor aspects related to the controller has come to be known as “dynamic density.” The words airspace complexity and dynamic density are used interchangeably in this paper.

Earlier studies show that dynamic density is highly correlated with sector controller activity levels and it can be used as a good indicator of controller workload [2]. Several efforts have been made to identify dynamic density measures that correlate well with workload in studied sectors with prevalent procedures [3]. Dynamic density models from different studies depend on the procedures, displays and the communications equipment used during the calibration of the workload. However, many of the factors affecting controller workload are common to all these models.

Sector capacities are used as constraints in the design of TFM algorithms. For dynamic density and other airspace complexity measures [4-7] to be useful as traffic management tools, it is necessary to predict them for durations of 30 to 120 minutes. Since dynamic density is a function of the position and velocity of all aircraft in a sector, a trajectory prediction algorithm can be used to predict dynamic density [8]. The demand in a sector varies due to uncertainties in aircraft departure times, unscheduled and cancelled departures and changes to aircraft trajectories to avoid congested areas and areas affected by bad weather. The errors in the sector count vary with the prediction interval and can be as high as six aircraft for long durations [9]. As a result, there are situations where the actual number of aircraft in a sector can exceed MAP by several aircraft.

With a few exceptions [10-11], there has been very little work in the study of errors in the prediction of dynamic density. When the expected demand in the sector is well below MAP, addition of one or two unexpected aircraft is likely to keep the overall aircraft count below MAP and the associated traffic

manageable. The uncertainties are more significant when a sector has high levels of traffic since the addition of one or more aircraft may increase the controller workload beyond what can be handled safely. Recently, reference [12] has proposed recalibrating dynamic density models based on the predictability of various components of the metric.

This paper takes a different approach to the prediction of dynamic density by examining the errors when the sector count is close to the MAP value. Thus, characterizing the impact of one or two additional unanticipated aircraft on traffic complexity of already congested sectors would be of significant value. Such impact is not uniform in all appearances of an additional aircraft, but may depend on the location and the intent of the unexpected aircraft relative to the expected aircraft. Knowledge of what is the magnitude of the worst-case impact and in what situations the impact would be worst would be useful in traffic flow management. Therefore, this study provides a fresh approach by focusing on the impact of additional unexpected aircraft on already congested sectors where managing the situation can be a problem.

The paper is organized as follows: The section on Airspace Complexity provides a status of the current research on air space complexity. The section on Forecasting Complexity provides an approach for dealing with airspace complexity in TFM applications by using some of the critical components of airspace complexity and studying its variation under perturbations. The Results section provides results based on this approach for a busy sector ZDC32 in the Washington Center. The next section looks at the generality of the observations for ZDC32 by considering the complexity behavior of ZID81, another sector in the U.S airspace. The last section presents conclusions based on the work reported in this paper.

Airspace Complexity

The capacity of airspace is limited by the controller's ability to perform the tasks to maintain a safe system. The controller's ability to perform the tasks depends on the combination of traffic, procedures, tools, and mental models used to perform the tasks. Currently, the capacity of a sector is measured in terms of the maximum number of aircraft that can be accommodated in a sector, a quantity referred to as Monitor Alert Parameter. The traffic load in a sector is maintained at a level below the capacity of the

airspace. Although easy to measure and understand, MAP is not a satisfactory measure of capacity. It has been suggested by the Radio Technical Commission on Aeronautics [1] that the monitor/alert function should be extended to include measures of sector complexity and controller workload. These measures should be based not only on the number of aircraft, but their relation to each other, airspace geometry and varying traffic flow conditions. This concept has come to be known as "dynamic density." It is also referred to as airspace complexity [3-5].

Research on dynamic density relates traffic complexity factors, such as number of aircraft, aircraft separation and others, to controller workload. Researchers have identified dynamic density measures of complexity that correlate well with workload in studied sectors with prevalent procedures [1-2]. It has been argued that traffic pattern is a significant factor in a controller's cognitive ability [4]. As described later in the paper, published models derived in specific studies do not correlate well with each other and, hence, the generality of these measures beyond studied sectors seems to be in doubt. Dynamic density research identifies factors that contribute to the structural or traffic complexity of the situation.

Recently, there have been efforts to generate simplified measures of airspace complexity [13]. These models use the principal components of earlier airspace complexity metrics. Three airspace complexity measures are used in this paper. The first utilizes airspace complexity, C1, as described in [2]. C1 is a linear function of the number of aircraft in the sector (N), number of aircraft with heading change (NH), number of aircraft with speed change (NS), number of aircraft with altitude change (NA) and number of aircraft with horizontal and vertical proximity parameters (S5, S10, S25, S40 and S70). Airspace complexity is a linear combination of the above factors, i.e.,

$$C1 = W_1 \cdot N + W_2 \cdot NH + W_3 \cdot NS + W_4 \cdot NA + W_5 \cdot S5 + W_6 \cdot S10 + W_7 \cdot S25 + W_8 \cdot S40 + W_9 \cdot S70.$$

The weights, (W1,...,W9) were validated in a real-time operational environment based on the input from a large group of controllers.

The second airspace complexity function, C2, is a simplified version of [3] based on the components that contribute the most to that measure: aircraft density (AD1), number of aircraft close to sector boundary

(BPR), number of aircraft in the sector (N) and sector volume (SVOL). Although C2 does not correlate well with C1, the principal components of C2, namely, AD1, BPR, N and SVOL correlate well with C1 with an r-squared value of .87. A third complexity function was computed by calibrating AD1, BPR, N and SVOL using C1. With new weights, this function, C3, is used as the third complexity metric.

The traffic situation in ZDC32, a sector in Washington Center and one of the busiest sectors in the country, was examined every minute for five days in 2006. A careful examination of the data resulted in 5744 aircraft configurations. The values of C1, C2, C3 and N were computed for each of these configurations. Figure 1 shows the distribution of the sector counts and the three complexity metrics for these traffic conditions. The distributions in Figure 1 are normalized and their critical values are shown in Table 1. The complexity metrics have a Poisson-like distribution. Thus, with C1 complexity measure, maximum complexity value in the data is 141. The value at 95th percentile was 82. Thus, most traffic situations are not necessarily complex. The complexity function C2 results in negative values for certain aircraft configurations. The negative values are not meaningful and result from the inapplicability of certain complexity functions under all conditions.

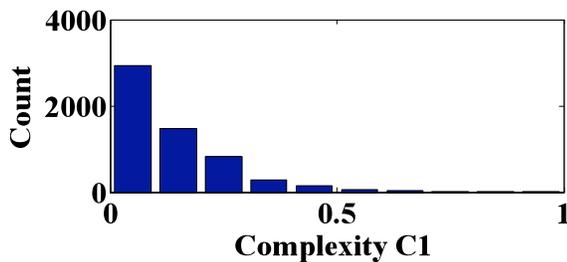


Figure 1.a Complexity C1 Distribution

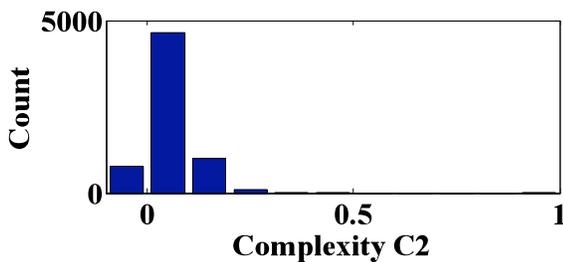


Figure 1.b Complexity C2 Distribution

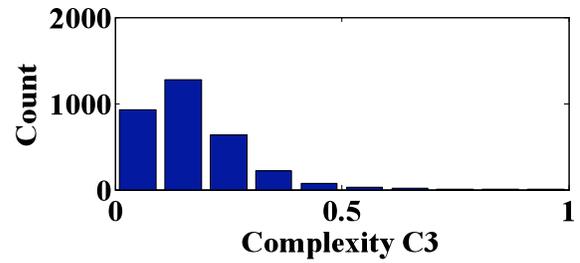


Figure 1.c Complexity C3 Distribution

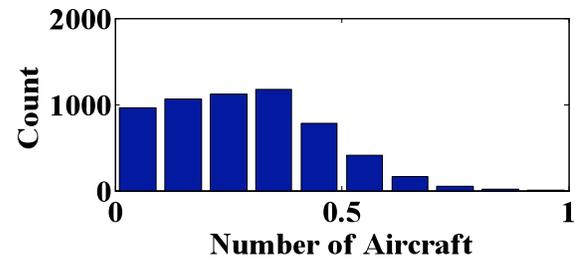


Figure 1.d Sector Count Distribution

Table 1. Critical Values of Metrics

Metric	Minimum	Maximum	95 th Percentile
C1	4	141	82
C2	-21	3.4	1.8
C3	4.7	86	51
N	4	18	12

Figure 2 shows the geometry and the aircraft configuration at a given time. The complexity values are 77 for C1, 2.4 for C2, 62.8 for C3, and the sector count N is 14. The complexity values correspond to 73%, 70%, 73% and 78% of the maximum values of these metrics.

Dynamic density and its principal components, although not precise, are good indicators of sector controller activity. For dynamic density to be useful, one should be able to predict its behavior. Since dynamic density is a function of the position and velocity of all aircraft in a sector, a trajectory prediction algorithm can be used to predict dynamic density. The trajectory-based models predict sector demand and dynamic density adequately for short

durations of up to twenty minutes. However, there are significant sources of error for predictions in the range of two hours. The accuracy of these predictions is impacted by aircraft modeling errors, lack of flight intent information, and departure and weather uncertainties [14-15].

Forecasting Complexity

The forecasting of dynamic density requires accurate position and velocity for all of the aircraft. As predicting even the correct sector count is hard, it is unreasonable to expect such information based on the current position and velocity of the aircraft and flight plan information. However, it is possible to predict the mean number of aircraft expected for a prediction interval of two hours based on historical data [16]. Assuming the mean sector count still leaves many possible configurations for the same number of aircraft. The paper uses a six step approach: (a) To emphasize the impact of errors in congested sectors, select one of the busiest sectors in the U.S. national airspace on days with high delay, (b) select traffic configurations with the mean number of aircraft expected for the prediction interval, (c) perturb the configurations by infusing one, two or three additional aircraft to the configurations and re-compute the airspace complexity metric, (d) group situations where the addition of a single aircraft increases complexity significantly, (e) identify interior regions and boundaries of the sector with significant impact, and (f) characterize the worst case impact locations. The approach thus aims to provide information of practical value. In general, the new aircraft can sometimes be an unscheduled flight (popup) that may appear at interior points or more often a re-routed or delayed aircraft that appears at sector boundaries. These cases will be studied separately.

The approach described earlier is examined using ZDC32 as an example in the next Section and provides insights into the impact of poor forecasting on complexity. This is followed by a Section that discusses application of the approach to another sector in Indianapolis Center demonstrating the general applicability of this approach.

Results for Washington Center Sector ZDC32

For simplicity, results presented in the rest of this paper utilize just one of these complexity measures,

C1. Similar analysis can be conducted for measures C2, C3 and others. As described earlier, traffic situations in ZDC32, a sector in Washington Center, were examined by the introduction of additional aircraft. ZDC32 is a high altitude sector in Washington Center. The primary function of ZDC is the separation of en route traffic and the sequencing of arrivals and departures of aircraft for the Washington-Baltimore Metropolitan Area, the New York Metropolitan Area, and Philadelphia.

The traffic situation in ZDC32 was examined for an additional six days for a total of eleven days in 2006. A list of locations where aircraft appear in ZDC32 was created and the resultant data consisted of 6550 aircraft configurations. Even though the exact aircraft configuration cannot be predicted as discussed earlier, it is instructive to examine the impact on a specific configuration. The final results in the paper are based on examining a good cross-section of all possible aircraft configurations. Therefore, the next subsection discusses the impact of a new aircraft on Dynamic Density of a specific aircraft configuration. This discussion is followed by the impact of additional aircraft as the number of aircraft in the configuration varies from a small number to MAP value.

Impact of an Additional Aircraft on a Specific Configuration

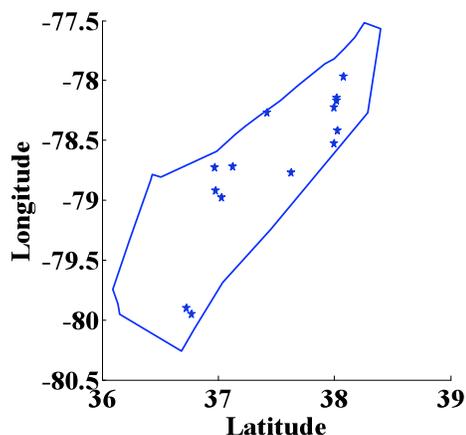


Figure 2. Aircraft Positions at 18:14

To study the impact of an additional aircraft on complexity, first consider a specific configuration with a given number of aircraft. Figure 2 shows the aircraft

configuration in ZDC32 at 18:14 Greenwich Mean Time (GMT) on July 21, 2006.

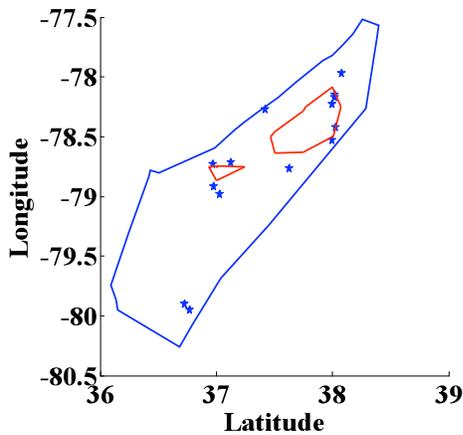


Figure 3. Contours Corresponding to C1= .96

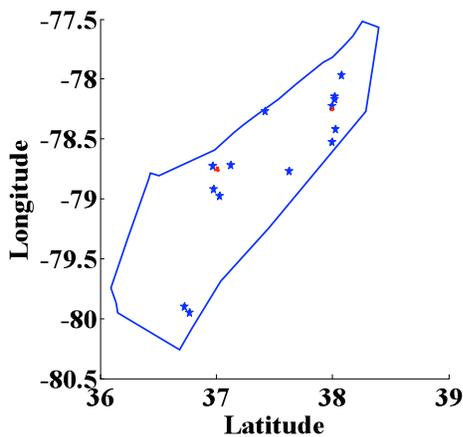


Figure 4. Contours Corresponding to C1=1.0

The C1 complexity value for this fourteen aircraft configuration is .73. The airspace complexity of ZDC32 was varied by the introduction of an additional aircraft at 18:14 GMT. It was found that the increase in complexity depends critically on the location of the new aircraft. To study the relation of the increase in complexity with location, the sector was divided into equal intervals in altitude, latitude and longitude. A new aircraft was introduced at each of these locations and the C1 complexity of the resulting fifteen aircraft configuration was computed in each case. For

simplicity, the new aircraft was assumed to be flying at the cruise speed and in level flight. This simplification results in a small error in the absolute values and with little impact on the conclusions. The location of the 15th aircraft corresponding to the maximum of the C1 values provides the aircraft location with the most impact on C1 complexity. Figure 3 shows the contour of aircraft positions (shown in red color), which increase the C1 airspace complexity measure from .73 to .96. Figure 4 shows two aircraft locations (shown in red) that increase C1 to a maximum value of 1.0. The figures show that the locations of the new aircraft where the increase in complexity is the worst are close to aircraft clusters.

Impact of an Unexpected Popup Aircraft

The previous section identified the contours of positions of worst complexity impact when the exact positions of the aircraft are known. The need for accurate position and speed information is modified in this section by assuming that the expected number of aircraft is known and the actual number may vary by as much as three additional aircraft. For N-aircraft configurations, the approach described in the Section on Forecasting Complexity was used to characterize the impact of new aircraft.

Table 2: Location of Aircraft Position for Maximum Impact on Complexity

Number of aircraft	Latitude	Longitude	Maximum observed complexity	Worst Position increase
15	36.8	-79.2	.94	.21
14	37.0	-79.5	.82	.22
12	36.7	-79.3	.65	.20
8	36.8	-79.5	.40	.18
4	36.4	-79.7	.18	.12
All	36.6	-79.5	1.0	.25

Table 2 shows the location of the new aircraft that increases the C1 complexity to its maximum value for different initial values of the number of aircraft in the sector. The data used had aircraft configurations varying from 4 to 18. MAP value for this sector is 18. The complexity of an aircraft configuration with 18 aircraft shown in the Figure 5 had the worst value of 141. All reported values are normalized with respect to this complexity value. The first row in the table describes the impact in the case of fifteen aircraft configurations. In these configurations, the worst-case

impact takes place at latitude of 36.8 and a longitude of -79.2 with the value of increase to be 0.21. Thus, .21 means that absolute value is 21% of the observed maximum value in the dataset.

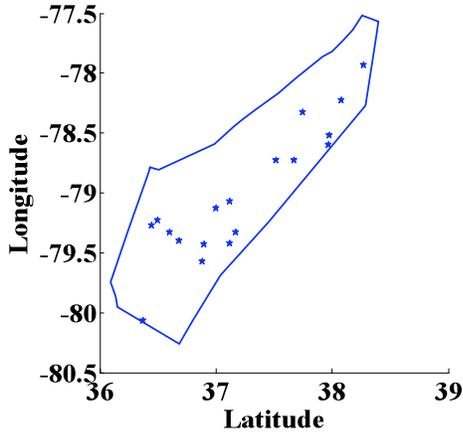


Figure 5. Most Complex Observed Configuration

The maximum complexity value of observed fifteen aircraft configuration is .94. Thus, the overall complexity after adding an aircraft in worst situation can reach as high as 1.15. The resultant situation can be 15% more complex than the worst situation observed in real data. This is clearly not a desirable situation. The second to fifth rows in the table lists entries for 14, 12, 8 and 4 aircraft. The last row lists results when all configurations are considered without limiting these to a specific number of aircraft in the configuration. If the number of aircraft in configurations considered is larger, then the corresponding worst-case impact is larger as well. As evident from the table, the situation resulting when one aircraft is added to 4, 8, or 12 aircraft configurations is of less concern as the worst complexity after adding an aircraft remains below 1. Most of the time, number of aircraft in a sector will be well below MAP value as can be seen in Figure 1.d and hence, the appearance of an unexpected aircraft does not result in excessive workload. While the points of worst impact in the Table 2 vary in latitude from 36.4 to 37 and in longitude from -79.2 to -79.7 depending on how many aircraft are in the configuration, all are in the neighborhood of each other. Also, these happen to be in the region with highest aircraft density where two major flows in ZDC32 intersect. Figure 6 below shows ZDC32 with the overall worst impact point as a small

black rectangle. Visually, one can see that this is in the region of the intersection of two major flows in the sector. One may also identify the worst impacted region. Figure 6 also shows a contour plot with 80th percentile values of impact on complexity. Again, this region is centered on the intersection of major flows.

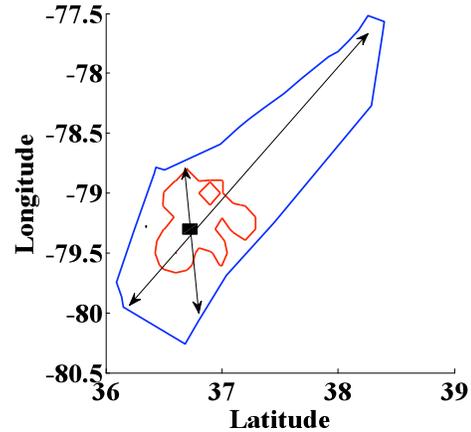


Figure 6. Worst Impact Contours for ZDC32

Impact of an Unexpected Aircraft at Sector Boundaries

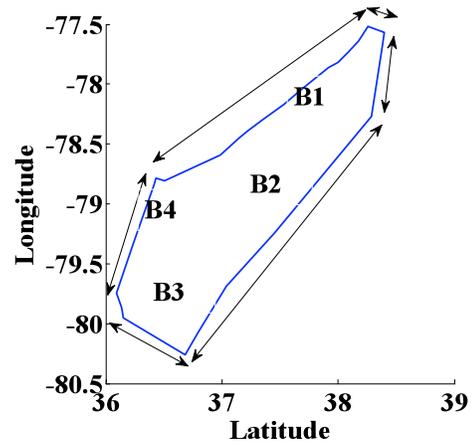


Figure 7. ZDC32 Boundaries

While popup aircraft and late departure aircraft can appear at interior points in the sector, rerouted new aircraft are more likely to appear at sector borders. Therefore, an attempt was made to identify border

segments, where an unexpected aircraft would make the worst impact. This was done by creating four boundary regions at ten flight levels, altitudes at intervals of 1000 feet, in ZDC32. The flight levels were selected in the range 24,000 to 33,000 feet. The four regions are shown as B1, B2, B3 and B4 in Figure 7. The worst possible impact varies significantly depending on the boundary. The variation is illustrated using two borders: (1) B1 at 31000 feet and (2) B4 at 25000 feet. While average probability of an aircraft entering at any of the forty boundaries in 10 flight levels is 2.5%, these are 5% for B1 at 31,000 feet and 1% for B4 at 25,000 feet.

As can be see from Table 3, for configurations with aircraft less than fifteen, the worst-case impact does not matter much as the worst-case complexity is already low. For aircraft configurations with fifteen aircraft, the boundary B1 at altitude 31,000 feet had an increase in complexity of 0.21, whereas the boundary B4 at altitude 25,000 feet had an increase in complexity of 0.05. Thus, there is significant variation in the impact on the complexity depending on the region of entry of the aircraft.

Table 3. Impact of Aircraft Entry Point on Complexity

Number of aircraft in configurations	Maximum observed complexity	Increase at B4 25,000 ft	Increase at B1 31,000 ft
15	.94	.05	.21
14	.82	.05	.20
12	.65	.05	.18
8	.40	.05	.16
4	.18	.05	.11
All	1.0	.05	.21

Impact of More Than One Unexpected Aircraft

The analysis for the impact of an additional aircraft on complexity can be repeated in the event that the actual number of aircraft in the prediction interval

of interest is off by two or three aircraft from the expected mean number of aircraft. The positions of worst impact at interior points are very similar to the single aircraft case. However, as can be seen from Table 4, the magnitude of the impact of introducing 2nd or 3rd aircraft is higher than in single aircraft case. If both the first and second aircraft appear at the worst positions, then the overall increase would be $0.25 + 0.29 = 0.54$.

Table 4. Location of Multiple Aircraft Positions in ZDC32 for Impact on Complexity

Number of aircraft introduced	Impact position	Impact value	Impact for B1 31,000 ft	Impact for B4 25,000 ft
1	36.6,79.5	.27	.25	.05
2	36.6, 79.5	.29	.28	.05
3	36.6, 79.5	.40	.32	.05

Just as B1 at 31,000 feet is the worst boundary for a single new aircraft as compared to B4 at 24,000 feet, it is also the worst boundary in the case of a second or a third new aircraft. Second or third aircraft appearing at B1 at 31,000 feet has worse impact as compared to a second or third aircraft appearing at B4 at 25,000 feet. The exact impact value is slightly worse than the impact value in the case of a single aircraft. Table 4 shows these values.

Results for ZID81

To examine the generality of the approach, this methodology was applied to another sector in Indianapolis Center. ZID81 is a high altitude sector in the southwest corner of Indianapolis Center. It deals with en route traffic and departures from Chicago O’Hare (ORD) and Detroit (DTW) airports The MAP value is seventeen. The three major flows in the sector are (1) east-west flights between Cincinnati and Saint Louis, (2) flights between Chicago, Detroit and Milwaukee airports to Florida destinations and (3) over-flights between Texas and Tennessee to east coast destinations.

The intersection of the three major flows is shown in the Fig. 8. The red contour in the figure indicates the 80th percentile of complexity. The worst impact point at interior locations, shown as an asterisk in Fig. 8, centers on latitude of 37.90 and longitude of -87.80 and is near the point of intersection of three major flows.

Like in the case of ZDC32, the impact of a new aircraft entering a sector at boundary depends on the entry boundary. Figure 9 shows four boundaries of the sector. The differential impact between boundaries can be illustrated with two boundaries, B3 at 31,000 feet and B1 at 28,000 feet. The probability of an aircraft entering B3 at FL310 is 12% whereas the probability of an aircraft entering B1 at 28,000 feet is 8%. As can be seen from Table 5, aircraft entering boundary B3 at 31,000 feet have much more impact than aircraft entering boundary B1 at 28,000 feet.

Table 5. Location of Multiple Aircraft Positions in ZID81 for Impact on Complexity

Number of unexpected aircraft	Worst Impact Position	B3 at FL310	B1 at FL280
1	37.9, -87.8	.19	.06
2	37.9, -87.8	.20	.06
3	37.9, -87.8	.21	.06

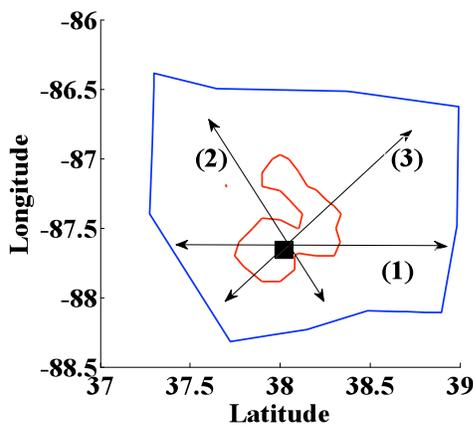


Figure 8. ZID81 Worst Contour Positions

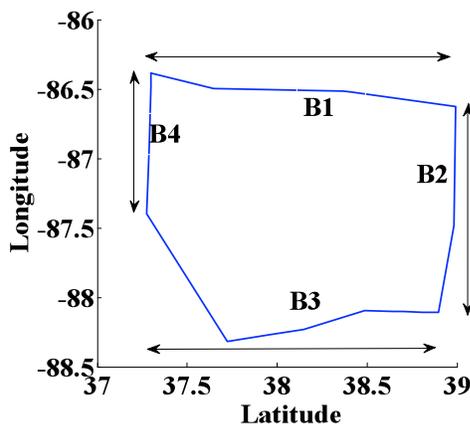


Figure 9. ZID81 Boundaries

Concluding Remarks

The forecasting of Dynamic Density metrics for prediction intervals of two hours and longer requires both the position and the velocity of all the aircraft. As predicting even the correct sector count is hard, it is unreasonable to expect correct position and velocity information derived from schedule or flight plan information. However, it is reasonable to assume the mean number of aircraft can be predicted for a prediction interval of two hours based on historical data, but there may be one, two or three additional aircraft. Given the nature of uncertainties of positions of the aircraft, one can question if Dynamic Density metrics can be of any value. The approach outlined in this paper shows that a useful characterization of expected complexity can be made even in the presence of such uncertainties.

The impact of additional aircraft on complexity is not uniform for all appearances of an additional aircraft, but depends on the location of the unexpected aircraft relative to the expected aircraft. Knowledge of what is the magnitude of worst-case impact and in what situations the impact would be worst would be useful in traffic flow management. New unexpected aircraft maybe popup aircraft that appear at interior points in a sector or aircraft that appear at boundaries of a sector. Regions of worst impact for popup aircraft were found to be the intersections of major flows in the case of ZDC32 and ZID81. Also, it was found that the impact of a new aircraft appearing at sector boundaries varies significantly depending on the boundary in the case of both sectors ZDC32 and ZID81.

This analysis methodology to characterize the prediction characteristics of dynamic density can be extended to other sectors and other models of dynamic density.

Depending on the sector and the complexity function, when a sector is highly congested, the method identifies aircraft entering the sector at certain locations, boundaries and altitudes, whose errors in prediction contribute significantly to the increase in workload. If these errors cannot be reduced, it may be necessary to limit the traffic approaching the sector from these altitudes and boundaries.

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